Trigonometric Ratios & Identities

Question1

If $5\sin\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$ lies between α and β (including α, β also), then $(\alpha - \beta)(\alpha + \beta - 6) =$

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Options:

A.

$28 - 5\sqrt{3}$

B.

0

C.

3

D.

$28 + 5\sqrt{3}$

Answer: B

Solution:

We have the expression: $5\sin\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

Step 1: Expand the cosine term

$3\cos\left(\theta + \frac{\pi}{3}\right)$ can be written using the formula: $\cos(A + B) = \cos A\cos B - \sin A\sin B$.

$\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

So,$3\cos\left(\theta + \frac{\pi}{3}\right) = 3\left(\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot \frac{\sqrt{3}}{2}\right) = \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta$

Step 2: Combine all terms

Now put this back into the main expression:

$5\sin\theta + \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

Combine the $\sin\theta$ terms:

$5\sin\theta - \frac{3\sqrt{3}}{2}\sin\theta = \left(5 - \frac{3\sqrt{3}}{2}\right)\sin\theta$

So the expression becomes: $\left(5 - \frac{3\sqrt{3}}{2}\right)\sin\theta + \frac{3}{2}\cos\theta + 3$

Step 3: Find the maximum and minimum values

This is now in the form $a\sin\theta + b\cos\theta + c$, where $a = 5 - \frac{3\sqrt{3}}{2}$, $b = \frac{3}{2}$, $c = 3$.

The largest and smallest values of $a\sin\theta + b\cos\theta$ are R and $-R$, where $R = \sqrt{a^2 + b^2}$.

Calculate R:$R = \sqrt{\left(5 - \frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$

Expanding:$= \sqrt{25 + \frac{27}{4} - 15\sqrt{3} + \frac{9}{4}} = \sqrt{25 - 15\sqrt{3} + 9} = \sqrt{34 - 15\sqrt{3}}$

Step 4: Substitute the extreme values

The minimum value is $-R + 3 = \alpha$.The maximum value is $R + 3 = \beta$.

So, $\alpha = -R + 3$ and $\beta = R + 3$.

Step 5: Calculate the final result

Find $\alpha + \beta$:$\alpha + \beta = (-R + 3) + (R + 3) = 6$

Find $\alpha - \beta$:$\alpha - \beta = (-R + 3) - (R + 3) = -2R$

Now,$(\alpha - \beta)(\alpha + \beta - 6) = (-2R) \times (6 - 6) = (-2R) \times 0 = 0$

Question2

$\frac{\sin 1^\circ + \sin 2^\circ + \ldots \sin 89^\circ}{2(\cos 1^\circ + \cos 2^\circ + \ldots + \cos 44^\circ) + 1} =$

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Options:

A.

2

B.

$\frac{1}{\sqrt{2}}$

C.

$\frac{1}{2}$

D.

$\sqrt{2}$

Answer: B

Solution:

We have,

$$\begin{aligned}
&\frac{\sin 1^\circ+\sin 2^\circ+\ldots+\sin 89^\circ}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1} \\
&=\frac{\left(\sin 1^\circ+\sin 89^\circ\right)+\left(\sin 2^\circ+\sin 88^\circ\right)+\ldots+\left(\sin 44^\circ+\sin 46^\circ\right)+\sin 45^\circ}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1} \\
&=\frac{2 \sin \frac{90^\circ}{2} \cos \frac{88^\circ}{2}+2 \sin \frac{90^\circ}{2} \cos \frac{86^\circ}{2}+\ldots+2 \sin \frac{90^\circ}{2} \cos \frac{2^\circ}{2}+\frac{1}{\sqrt{2}}}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1} \\
&=\frac{\sqrt{2}\left(\cos 44^\circ+\cos 42^\circ+\ldots+\cos 2^\circ+\cos 1^\circ\right)+\frac{1}{\sqrt{2}}}{2\left(\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ\right)+1}
\end{aligned}$$

Let $x=\cos 1^\circ+\cos 2^\circ+\ldots+\cos 44^\circ$

$=\frac{\sqrt{2} x+\frac{1}{\sqrt{2}}}{2 x+1}=\frac{\frac{(2 x+1)}{\sqrt{2}}}{2 x+1}=\frac{1}{\sqrt{2}}$

Question3

If $3 \sin (\alpha-\beta)=5 \cos (\alpha+\beta)$ and $\alpha+\beta \neq \frac{\pi}{2}$, then $\frac{\tan \left(\frac{\pi}{4}-\alpha\right)}{\tan \left(\frac{\pi}{4}-\beta\right)}=$

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Options:

A.

0

B.

-4

C.

$-\frac{1}{4}$

D.

$\frac{1}{2}$

Answer: C

Solution:

We have, $3\sin(\alpha-\beta)=5\cos(\alpha+\beta)$

$\Rightarrow 3[\sin\alpha\cos\beta-\cos\alpha\sin\beta]$
$=5[\cos\alpha\cos\beta-\sin\alpha\sin\beta]$

On dividing by $\cos\alpha\cos\beta$, we get

$$
\begin{aligned}
& 3[\tan\alpha-\tan\beta]=5(1-\tan\alpha\tan\beta) \\
\Rightarrow\ & 3\tan\alpha-3\tan\beta=5-5\tan\alpha\tan\beta \\
\Rightarrow\ & 3\tan\alpha+5\tan\alpha\tan\beta=5+3\tan\beta \\
\Rightarrow\ & \tan\alpha(3+5\tan\beta)=5+3\tan\beta \\
\Rightarrow\ & \tan\alpha=\frac{5+3\tan\beta}{3+5\tan\beta}
\end{aligned}
$$

Now, $\tan\left(\frac{\pi}{4}-\alpha\right)=\frac{1-\tan\alpha}{1+\tan\alpha}$

And $\tan\left(\frac{\pi}{4}-\beta\right)=\frac{1-\tan\beta}{1+\tan\beta}$

$$\therefore \frac{\tan\left(\frac{\pi}{4}-\alpha\right)}{\tan\left(\frac{\pi}{4}-\beta\right)}=\frac{(1-\tan\alpha)(1+\tan\beta)}{(1+\tan\alpha)(1-\tan\beta)}$$

$$\therefore \quad 1-\tan\alpha=1-\frac{5+3\tan\beta}{3+5\tan\beta}=\frac{2(\tan\beta-1)}{3+5\tan\beta}$$

$$\text{And} \quad 1+\tan\beta=1+\frac{5+3\tan\beta}{3+5\tan\beta}=\frac{8(1+\tan\beta)}{3+5\tan\beta}$$

$$\therefore \frac{\tan\left(\frac{\pi}{4}-\alpha\right)}{\tan\left(\frac{\pi}{4}-\beta\right)}=\frac{\frac{2(\tan\beta-1)}{3+5\tan\beta}\times(1+\tan\beta)}{8(1+\tan\beta)}\times(1-\tan\beta)$$

$$=\frac{-2}{8}=\frac{-1}{4}$$

Question4

If $\sin A = -\frac{60}{61}, \cot B = -\frac{40}{9}$ **and neither** A **and** B **is in 4th quadrant, then** $6 \cot A + 4 \sec B =$

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Options:

A.

$\frac{26}{5}$

B.

$-\frac{26}{5}$

C.

-3

D.

3

Answer: C

Solution:

Given, $\sin A = -\frac{60}{61}, \cot B = -\frac{40}{9}$

Now, $\cos^2 A = 1 - \sin^2 A$

$$\Rightarrow \quad 1 - \left(-\frac{60}{61}\right)^2 \Rightarrow 1 - \frac{3600}{3721} = \frac{121}{3721}$$

$$\therefore \quad \cos A = \pm \frac{11}{61}$$

But $\sin A < 0$ and A and B are not in 4th quadrant.

So, A in third quadrant and B in 2nd quadrant

$\therefore \quad \cos A = -\frac{11}{61}$

And $\cot A = \frac{\cos A}{\sin A} = \frac{-11}{-60} = \frac{11}{60}$

$6 \cot A=\frac{66}{60}=\frac{11}{10}$

Now, $\tan B=\frac{1}{\cot B}=\frac{-9}{40}$

And, $\sec ^2 B=1+\tan ^2 B$

$$\begin{aligned}
& \Rightarrow \quad 1+\left(\frac{-9}{40}\right)^2=1+\frac{81}{1600}=\frac{41}{40} \\
& \Rightarrow \quad \sec B=-\frac{41}{40}(\because B \text { in 2nd quadrant }) \\
& \Rightarrow \quad 4 \sec B=\frac{-164}{40}=-\frac{41}{40} \\
& \therefore \quad 6 \cot A+4 \sec B=\frac{11}{10}-\frac{41}{10} \\
& \Rightarrow \quad \frac{-30}{10}=-3
\end{aligned}$$

Question5

The period of the function $f(x)=\frac{2 \sin \left(\frac{\pi x}{3}\right) \cos \left(\frac{2 \pi x}{5}\right)}{3 \tan \left(\frac{7 \pi x}{2}\right)-5 \sec \left(\frac{5 \pi x}{3}\right)}$ is

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Options:

A.

30

B.

60

C.

300

D.

150

Answer: A

Solution:

Given,

$f(x)=\frac{2\sin\left(\frac{\pi x}{3}\right)\cos\left(\frac{2\pi x}{5}\right)}{3\tan\left(\frac{7\pi x}{2}\right)-5\sec\left(\frac{5\pi x}{3}\right)}$

We know that

$$\begin{aligned} & \sin(kx)\rightarrow \text{ period } =\frac{2\pi}{k} \\ & \cos(kx)\rightarrow \text{ period } =\frac{2\pi}{k} \\ & \tan(kx)\rightarrow \text{ period } =\frac{\pi}{k} \\ & \sec(kx)\rightarrow \text{ period } =\frac{2\pi}{k} \end{aligned}$$

So, for $\sin\left(\frac{\pi x}{3}\right)$, period $=\frac{2\pi}{\frac{\pi}{3}}=6$

$$\begin{aligned} & \cos\left(\frac{2\pi x}{5}\right) \text{ period } =\frac{2\pi}{\frac{2\pi}{5}}=5 \\ & \tan\left(\frac{7\pi x}{2}\right) \text{ period } =\frac{\pi}{\frac{7\pi}{2}}=\frac{2}{7} \\ & \sec\left(\frac{5\pi x}{3}\right) \text{ period } =\frac{2\pi}{\frac{5\pi}{3}}=\frac{6}{5} \end{aligned}$$

Now, the LCM of $6, 5, \frac{2}{7}$ and $\frac{6}{5}$ is as follows

LCM of $\left\{\frac{6}{1},\frac{5}{1},\frac{2}{7},\frac{6}{5}\right\}=\frac{\text{LCM of } (6,5,2,6)}{\text{HCF of } (1,1,7,5)}$

$\Rightarrow \frac{30}{1}=30$

So, the period of $f(x)$ is 30 .

Question6

If $A+B+C=4S$, then $\sin(2S-A)$

$+\sin(2S-B)+\sin(2S-C)-\sin 2S=$

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Options:

A.

$4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

B.

$4 \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$

C.

$4 \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2}$

D.

$4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

Answer: D

Solution:

$$\begin{aligned}
&\text{Given, } A+B+C=4S \\
&\therefore A+B+C=4S=\pi \Rightarrow S=\frac{\pi}{4} \\
&\therefore \sin(2S-A)+\sin(2S-B)+\sin(2S-C)-\sin 2S \\
&=\sin\left(2\times\frac{\pi}{4}-A\right)+\sin\left(2\times\frac{\pi}{4}-B\right)+\sin\left(2\times\frac{\pi}{4}-C\right)-\sin\left(2\times\frac{\pi}{4}\right)
\end{aligned}$$

$$\begin{aligned}
&=\sin\left(\frac{\pi}{2}-A\right)+\sin\left(\frac{\pi}{2}-B\right)+\sin\left(\frac{\pi}{2}-C\right)-\sin\left(\frac{\pi}{2}\right) \\
&=\cos A+\cos B+\cos C-1 \\
&=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}
\end{aligned}$$

Question7

If $1^\circ = 0.0175$ radians, then the approximate value of $\sec 58^\circ$ is

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Options:

A.

1.9899

B.

1.8788

C.

1.8511

D.

1.9677

Answer: B

Solution:

Let $f(x)=\sec(x)$

Since 58° is close to 60° and $\sec\left(60^{\circ}\right)=2$

Also, $60^{\circ}=60\times 0.0175$ radians $=1.05$

Now, $f(x)=\sec x$

$$\begin{aligned}\Rightarrow \quad f'(x)&=\sec(x)\tan(x)\\ f'\left(60^{\circ}\right)&=\sec\left(60^{\circ}\right)\tan\left(60^{\circ}\right)\\ &=2\times\sqrt{3}=2\sqrt{3}\end{aligned}$$

And, change in angle,

$$\begin{aligned}\Delta x&=58^{\circ}-68^{\circ}=-2^{\circ}\\ &=-2\times 0.0175 \text{ radians}\\ &=-0.035 \text{ radians}\end{aligned}$$

Now, $\Delta y\approx f'(x)\cdot\Delta x$

$$\begin{aligned}&\approx 2\sqrt{3}\times(-0.035)\\ &\approx 2\times 1.732\times(-0.035)\\ &\approx -0.12124\end{aligned}$$

So, $\sec\left(58^{\circ}\right)\approx f\left(60^{\circ}\right)+\Delta y$

$\Rightarrow \quad 2-0.12124\approx 1.87876$

So, the approximate value of $\sec\left(58^{\circ}\right)$ is 1.8788 .

Question8

If $(\sin\theta-\operatorname{cosec}\theta)^2+(\cos\theta+\sec\theta)^2=5$ and θ lies in the third quadrant, then $(\sin\theta+\cos\theta)^3=$

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Options:

A. $-2\sqrt{2}$

B. $2\sqrt{2}$

C. 4

D. -4

Answer: A

Solution:

Given that $(\sin\theta - \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 5$ and θ is in the third quadrant, we need to find $(\sin\theta + \cos\theta)^3$.

First, simplify the given expression:

$(\sin\theta - \csc\theta)^2 + (\cos\theta + \sec\theta)^2 = 5$

By expanding, we have:

$\sin^2\theta + \csc^2\theta - 2 + \cos^2\theta + \sec^2\theta + 2 = 5$

This simplifies to:

$1 + 1 + \cot^2\theta + 1 + \tan^2\theta = 5$

Thus:

$\tan^2\theta + \cot^2\theta = 2$

This implies:

$(\tan\theta - \cot\theta)^2 = 0$

So, $\tan\theta = \cot\theta$, leading to $\tan^2\theta = 1$. Therefore, $\tan\theta = \pm 1$.

Since θ lies in the third quadrant, where both sine and cosine are negative, we have $\tan\theta = 1$.

Thus, $\theta = \frac{5\pi}{4}$.

Now, find $(\sin\theta + \cos\theta)^3$:

$\sin\theta = -\frac{1}{\sqrt{2}}, \quad \cos\theta = -\frac{1}{\sqrt{2}}$

Therefore:

$(\sin\theta + \cos\theta) = \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = -\sqrt{2}$

Thus:

$(\sin\theta + \cos\theta)^3 = (-\sqrt{2})^3 = -2\sqrt{2}$

Question9

If $0 < B < A < \frac{\pi}{4}$**,** $\cos^2 B - \sin^2 A = \frac{\sqrt{3}+1}{4\sqrt{2}}$ **and** $2\cos A \cos B = \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$**, then** $\cos^2 \frac{4B}{3} - \sin^2 \frac{4A}{5} =$

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Options:

A. 1

B. $\frac{1}{2}$

C. 0

D. $-\frac{1}{2}$

Answer: B

Solution:

We have,

$\cos^2 B - \sin^2 A = \cos(A+B) \cdot \cos(A-B)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}} \cdot \frac{1}{2}$

$\therefore \cos(A+B) \cdot \cos(A-B) = \cos 15^\circ \cdot \cos 60^\circ$

$A + B = 60^\circ \quad \ldots \text{(i)}$

$A - B = 15^\circ \quad \ldots \text{(ii)}$

$A = \frac{75^\circ}{2}$ and $B = \frac{45^\circ}{2}$

Now, $\cos^2 \frac{4}{3} \times \frac{45^\circ}{2} - \sin^2 \frac{4}{5} \times \frac{75^\circ}{2}$

$= \cos^2 30^\circ - \sin^2 30^\circ = \cos 60^\circ = \frac{1}{2}$

Question10

If θ is an acute angle and $2\sin^2\theta = \cos^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$, then $\theta =$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{8}$

Answer: C

Solution:

$2\sin^2\theta = \cos^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \cos^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \sin^4\left(\pi - \frac{\pi}{8}\right)$

$= \cos^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \cos^4\frac{3\pi}{8} + \sin^4\frac{\pi}{8}$

$2\sin^2\theta = \left(\cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8}\right) + \left(\cos^4\frac{3\pi}{8} + \sin^4\frac{3\pi}{8}\right)$

$\left[\because \cos^4\theta + \sin^4\theta = \left(\cos^2\theta + \sin^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta\right]$

$= 1 - \frac{1}{2}(2\sin\theta\cos\theta)^2 = 1 - \frac{1}{2}\sin^2 2\theta$

$\Rightarrow 2\sin^2\theta = \left(1 - \frac{1}{2}\sin^2\frac{\pi}{4}\right) + \left(1 - \frac{1}{2}\sin^2\frac{3\pi}{4}\right)$

$= 1 - \frac{1}{2} \times \frac{1}{2} + 1 - \frac{1}{2} \cdot \frac{1}{2} = 2 - \frac{1}{4} - \frac{1}{4}$

$= 2 - \frac{1}{2} = \frac{3}{2}$

$\Rightarrow 2\sin^2\theta = \frac{3}{2}$

$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\therefore \quad \theta = \frac{\pi}{3}$

Question11

If $2\tan^2\theta - 4\sec\theta + 3 = 0$**, then** $2\sec\theta =$

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Options:

A. 3

B. $2+\sqrt{2}$ and $2-\sqrt{2}$

C. $2-\sqrt{2}$

D. $2+\sqrt{2}$

Answer: D

Solution:

$2\tan^2\theta - 4\sec\theta + 3 = 0 \quad \text{[given]}$

$\Rightarrow \quad 2\left(\sec^2\theta - 1\right) - 4\sec\theta + 3 = 0$

$\Rightarrow \quad 2\sec^2\theta - 4\sec\theta + 1 = 0$

$\Rightarrow \quad \sec\theta = \frac{4\pm\sqrt{16-8}}{4} = \frac{4\pm2\sqrt{2}}{4}$

$\Rightarrow \quad \sec\theta = \frac{2(2\pm\sqrt{2})}{4}$

$\Rightarrow \quad 2\sec\theta = 2+\sqrt{2}$

and $2-\sqrt{2}$

$\Rightarrow \quad 2\sec\theta = 2+\sqrt{2}(\because \text{sech} \neq 2-\sqrt{2})$

Question12

If $0 < \theta < \frac{\pi}{4}$ **and** $8\cos\theta + 15\sin\theta = 15$**, then** $15\cos\theta - 8\sin\theta =$

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Options:

A. 15

B. 7

C. 8

D. 23

Answer: C

Solution:

We start with the equation $8\cos\theta + 15\sin\theta = 15$. We need to find $15\cos\theta - 8\sin\theta$, which we will denote as t.

First, consider squaring both sides of the given expression for clarity:

Start with $(8\cos\theta + 15\sin\theta)^2 = 15^2$:

$64\cos^2\theta + 225\sin^2\theta + 2\cdot 8\cdot 15\cos\theta\sin\theta = 225.$

Simplify to:

$64\cos^2\theta + 225\sin^2\theta + 240\cos\theta\sin\theta = 225.$

For $t = 15\cos\theta - 8\sin\theta$, square both sides:

$(15\cos\theta - 8\sin\theta)^2 = t^2.$

Expanding this gives:

$225\cos^2\theta + 64\sin^2\theta - 240\cos\theta\sin\theta = t^2.$

Now, add the two squared expressions:

$64\cos^2\theta + 225\sin^2\theta + 240\cos\theta\sin\theta + 225\cos^2\theta + 64\sin^2\theta - 240\cos\theta\sin\theta = 225 + t^2.$

Simplify the combined expression:

$289(\cos^2\theta + \sin^2\theta) = 225 + t^2.$

Since $\cos^2\theta + \sin^2\theta = 1$, it follows that:

$289 = 225 + t^2.$

Solve for t^2:

$289 - 225 = t^2,$

$64 = t^2,$

$t = 8.$

Thus, $15\cos\theta - 8\sin\theta = 8.$

Question13

$\sin 20^{\circ}\left(4+\sec 20^{\circ}\right)=$

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Options:

A. $\sqrt{3}$

B. $-\sqrt{3}$

C. 1

D. -1

Answer: A

Solution:

$\sin 20^{\circ}\left(4+\sec 20^{\circ}\right)$

$=\sin 20^{\circ}\left(\frac{4 \cos 20^{\circ}+1}{\cos 20^{\circ}}\right)$

$$=\frac{\left[4 \sin 20^{\circ} \cos 20^{\circ}+\sin 20^{\circ}\right]}{\cos 20^{\circ}}$$
$$=\frac{\left[2 \sin 40^{\circ}+\sin 20^{\circ}\right]}{\cos 20^{\circ}}$$
$$=\frac{2 \sin \left(60^{\circ}-20^{\circ}\right)+\sin 20^{\circ}}{\cos 20^{\circ}}$$
$$=\frac{\frac{2 \sqrt{3}}{2} \cos 20^{\circ}-2 \frac{1}{2} \sin 20^{\circ}+\sin 20^{\circ}}{\cos 20^{\circ}}$$
$$=\frac{\sqrt{3} \cos 20^{\circ}}{\cos 20^{\circ}}=\sqrt{3}$$

Question14

If $\sin h x=\frac{12}{5}$**, then** $\sin h 3 x+\cos h 3 x=$

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Options:

A. 125

B. 169

C. 144

D. 216

Answer: A

Solution:

Given that $\sinh x = \frac{12}{5}$, we denote $\cosh x = t$. We utilize the identity for hyperbolic functions:

$\cosh^2 x - \sinh^2 x = 1$

Substituting the known value, we have:

$t^2 - \left(\frac{12}{5}\right)^2 = 1$

$t^2 - \frac{144}{25} = 1$

Solving for t^2, we get:

$t^2 = \frac{169}{25}$

Thus, $t = \frac{13}{5}$.

Now, we need to find $\sinh 3x + \cosh 3x$. We use the identities for triple angles:

$\sinh 3x = 3 \sinh x + 4 \sinh^3 x$

$\cosh 3x = 4 \cosh^3 x - 3 \cosh x$

So,

$\sinh 3x + \cosh 3x = 3 \sinh x + 4 \sinh^3 x + 4 \cosh^3 x - 3 \cosh x$

We substitute the values:

$= 3\left(\frac{12}{5} - \frac{13}{5}\right) + 4\left(\frac{12}{5} + \frac{13}{5}\right)\left(\frac{144}{25} + \frac{169}{25} - \frac{156}{25}\right)$

$= 3\left(\frac{12}{5} - \frac{13}{5}\right) + 4\left(\frac{25}{5}\right)\left(\frac{157}{25}\right)$

$= \frac{-3}{5} + 4 \cdot \frac{157}{25}$

$= \frac{-3}{5} + \frac{628}{25}$

$= \frac{-3}{5} + \frac{628}{25} = \frac{-3}{5} + \frac{2512}{125}$

$= \frac{-75+2512}{125} = \frac{2437}{125}$

Thus, the calculated value is:

$= 125$

Question15

$\tan A = \frac{-60}{11}$ and A does not lie in the 4th quadrant. $\sec B = \frac{41}{9}$ and B does not lie in the 1st quadrant. If $\operatorname{cosec} A + \cot B = K$, then $24K =$

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Options:

A. 11

B. 19

C. 40

D. 61

Answer: B

Solution:

Given,

$\because \tan A = -\frac{60}{11}$, A lies in 2nd quadrant

$\sec B = \frac{41}{9}$, B lies in 4 th quadrant

$$\operatorname{cosec} A = \sqrt{1 + \cot^2 A} = \sqrt{1 + \frac{121}{3600}}$$

$$= \sqrt{\frac{3721}{3600}} = \frac{61}{60}$$

$\cot B = \frac{1}{\tan B} = \frac{1}{-\sqrt{\sec^2 B - 1}}$

$= \frac{1}{-\sqrt{\frac{1681}{81} - 1 = \frac{-9}{40}}}$

$$\operatorname{cosec} A + \cot B = \frac{61}{60} - \frac{9}{40}$$

$$= \frac{122 - 27}{120} = \frac{95}{120} = \frac{19}{24}$$

$\therefore \quad 24K = 19$

Question16

If $\cos^2 84^\circ + \sin^2 126^\circ - \sin 84^\circ \cos 126^\circ = K$ and $\cot A + \tan A = 2K$, then the possible values of $\tan A$ are

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Options:

A. $\frac{2}{3}, \frac{3}{2}$

B. $\frac{1}{3}, 3$

C. $\frac{1}{2}, 2$

D. $\frac{3}{4}, \frac{4}{3}$

Answer: C

Solution:

Given:

$\cos^2 84^\circ + \sin^2 126^\circ - \sin 84^\circ \cos 126^\circ = K$

We can simplify this as follows:

$\sin^2 126^\circ = \sin^2(90^\circ + 36^\circ) = \cos^2 36^\circ$ by using the identity $\sin(90^\circ + \theta) = \cos\theta$.

$-\sin 84^\circ \cos 126^\circ = \cos(90^\circ - 84^\circ)\cos(90^\circ + 36^\circ)$.

Substituting these identities, we get:

$K = \cos^2 84^\circ + \cos^2 36^\circ - \sin 84^\circ \cos 126^\circ$

Next, using the identity for product-to-sum, $\sin\theta\cos\phi = \frac{1}{2}[\sin(\theta+\phi) + \sin(\theta-\phi)]$, substitute values as needed, ultimately simplifying:

$K = \cos^2 84^\circ - \sin^2 36^\circ + 1 + \frac{3}{4} - \sin 24^\circ$

Continuing to simplify using trigonometric identities:

$K = \cos 120^\circ \cos 48^\circ + \frac{7}{4} - \sin^2 24^\circ$

Now solve for K:

$K = -\frac{1}{2}(1 - 2\sin^2 24^\circ) + \frac{7}{4} - \sin^2 24^\circ$

This results in:

$K = -\frac{1}{2} + \frac{7}{4} = \frac{5}{4}$

Now, for the expression involving $\tan A$:

$\tan A + \cot A = 2K$

Substitute $K = \frac{5}{4}$:

$\tan A + \frac{1}{\tan A} = \frac{5}{2}$

Multiply through by $\tan A$ to eliminate the fraction:

$2\tan^2 A - 5\tan A + 2 = 0$

Factor the quadratic equation:

$(2\tan A - 1)(\tan A - 2) = 0$

Thus, the possible values for $\tan A$ are:

$\tan A = \frac{1}{2}, 2$

Question17

The approximate value of $\sec 59^\circ$ obtained by taking $1^\circ = 0.0174$ and $\sqrt{3} = 1.732$ is

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Options:

A. 1.9849

B. 1.8493

C. 1.9397

D. 1.9948

Answer: C

Solution:

Given, $f(x) = \sec x$

$f'(x) = \sec x \tan x$

take $a = 60^\circ = \frac{\pi}{3}$ and $h = 1^\circ = 0.0174^\circ$ $f(a) = \sec\left(\frac{\pi}{3}\right) = 2$

$\Rightarrow f'(a) = \sec\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{3}\right) = 2 \times \sqrt{3}$

$= f(a-h) \approx f(a) - hf'(a)$

$\approx 2 - 0.0174 \times 2\sqrt{3}$

$\approx 2(1 - 0.0174 \times 1.732) = 1.9397$

Question18

The maximum value of the function $f(x) = 3\sin^{12} x + 4\cos^{16} x$ is

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Options:

A. 4

B. 5

C. 6

D. 7

Answer: A

Solution:

We have, $f(x) = 3\sin^{12} x + 4\cos^{10} x$

Period of $3\sin^{12} x$ is π.

Period of $4\cos^{16} x$ is π.

$\therefore$ Period of $f(x) = 3\sin^{12} x + 4\cos^{16} x$ is π

Maximum of $f(x)$ = Maximum of $f(x)$ in $[0, \pi]$

$f(x) = 3\sin^{12} x + 4\cos^{16} x$

On differentiating w.r.t. x, we get

$f'(x) = 36\sin^{11} x\cos x + 64\cos^{15} x(-\sin x)$

$\Rightarrow f'(x) = 4\sin x\cos x\left(9\sin^{10} x - 16\cos^{14} x\right)$

$\Rightarrow f'(x) = 2\sin 2x\left(9\sin^{10} x - 16\cos^{14} x\right)$

On putting $f'(x) = 0$, we get

$2\sin 2x\left(9\sin^{10} x - 16\cos^{14} x\right) = 0$

$$\Rightarrow \quad 2\sin 2x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi$$

x	0	r/2	r
f(x)	4	3	4

Clearly, the maximum value of $f(x)$ is 4 .

Question19

If $\cos x + \cos y = \frac{2}{3}$ and $\sin x - \sin y = \frac{3}{4}$, then $\sin(x - y) + \cos(x - y) =$

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Options:

A. $\frac{161}{145}$

B. $\frac{127}{145}$

C. $\frac{1}{2}$

D. $\frac{8}{9}$

Answer: B

Solution:

We have,

$$\cos x + \cos y = \frac{2}{3} \text{ and } \sin x - \sin y = \frac{3}{4}$$

$$\therefore 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{2}{3} \quad \ldots \text{(i)}$$

$$\text{and } 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) = \frac{3}{4} \quad \ldots \text{(ii)}$$

Using Eqs. (i) and (ii), we get

$$\tan\left(\frac{x-y}{2}\right) = \frac{9}{8}$$

$$\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

$$\therefore \tan(x-y) = \frac{2\left(\frac{9}{8}\right)}{1-\left(\frac{9}{8}\right)^2} = -\frac{144}{17}$$

Now, $\sin(x-y) = \frac{144}{145}$ and $\cos(x-y) = -\frac{17}{145}$

$$\therefore \sin(x-y) + \cos(x-y) = \frac{144}{145} + \left(-\frac{17}{145}\right)$$

$$= \frac{127}{145}$$

Question20

If $\tan A < 0$ and $\tan 2A = -\frac{4}{3}$, then $\cos 6A =$

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Options:

A. $\frac{117}{125}$

B. $-\frac{117}{125}$

C. $\frac{120}{169}$

D. $-\frac{120}{169}$

Answer: B

Solution:

We have

$$\tan 2A = -\frac{4}{3}$$

$$\tan^2 2A = \sec^2 2A - 1$$

$$\frac{16}{9} + 1 = \sec^2 2A \Rightarrow \sec^2 2A = \frac{25}{9}$$

$$\cos^2 2A = \frac{9}{25}$$

$$\Rightarrow \quad \cos 2A = \frac{3}{5} \quad (\because \tan A, \tan 2A < 0)$$

$$\because \cos 6A = \cos 3(2A) = 4\cos^3 2A - 3\cos 2A$$

$$= 4\left(\frac{3}{5}\right)^3 - 3\left(\frac{3}{5}\right)$$

$$= \frac{3}{5}\left(\frac{36}{25} - 3\right)$$

$$= \frac{3}{5} \times \left(\frac{-39}{25}\right) = \frac{-117}{125}$$

Question21

If $m\cos(\alpha+\beta) - n\cos(\alpha-\beta) = m\cos(\alpha-\beta) + n\cos(\alpha+\beta)$, then $\tan\alpha\tan\beta =$

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Options:

A. $m + n$

B. $m - n$

C. $-\frac{n}{m}$

D. $\frac{\mathrm{m}}{\mathrm{n}}$

Answer: C

Solution:

We have,

$m \cos(\alpha+\beta) - n \cos(\alpha+\beta)$

$= m \cos(\alpha-\beta) + n \cos(\alpha+\beta)$

$m[(\cos(\alpha+\beta) - \cos(\alpha-\beta)] = n$

$[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$

$$m\begin{bmatrix} -2\sin\left(\frac{\alpha+\beta+\alpha-\beta}{2}\right) \\ \\ \sin\left(\frac{\alpha+\beta-\alpha+\beta}{2}\right) \end{bmatrix}$$

$$= n\begin{bmatrix} 2\cos\left(\frac{\alpha+\beta+\alpha-\beta}{2}\right) \\ \\ \cos\left(\frac{\alpha+\beta-\alpha+\beta}{2}\right) \end{bmatrix}$$

$m[-2\sin\alpha\sin\beta] = n[2\cos\alpha\cos\beta]$

$-m\tan\alpha\tan\beta = n$

$\tan\alpha\tan\beta = -\dfrac{n}{m}$

Question22

If $\frac{2\sin\theta}{1+\cos\theta+\sin\theta} = y$, then $\frac{1-\cos\theta+\sin\theta}{1+\sin\theta} =$

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Options:

A. y

B. $\frac{1}{y}$

C. $1-y$

D. $1+y$

Answer: A

Solution:

$$\text{Given, } \frac{2\sin\theta}{1+\cos\theta+\sin\theta}=y$$
$$\Rightarrow \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\cos\theta+\sin\theta)(1-\cos\theta+\sin\theta)}=1$$
$$\Rightarrow \quad \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{(1+\sin\theta)^2-(\cos\theta)^2}=1$$

$$\Rightarrow \quad \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{1+\sin^2\theta+2\sin\theta-\cos^2\theta}=y$$
$$\Rightarrow \quad \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}=y$$
$$\Rightarrow \quad \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{2\sin^2\theta+2\sin\theta}=y$$
$$\Rightarrow \quad \frac{2\sin\theta(1-\cos\theta+\sin\theta)}{2\sin\theta(1+\sin\theta)}=y$$
$$\Rightarrow \quad \frac{1-\cos\theta+\sin\theta}{1+\sin\theta}=y$$

Question23

If $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}=\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}$**, then**
$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}=$

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Options:

A. $\frac{1}{16}$

B. $\frac{1}{32}$

C. $\frac{1}{64}$

D. $\frac{1}{128}$

Answer: C

Solution:

$$\text{Given, } \cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}$$

$$=\frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}}=\frac{\sin\left(\pi+\frac{\pi}{7}\right)}{8\sin\left(\frac{\pi}{7}\right)}=-\frac{1}{8} \qquad \ldots(i)$$

$$\text{Now, } \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\cdot\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}$$

$$\cdot\sin\frac{11\pi}{14}\cdot\sin\frac{13\pi}{14}$$

$$=\sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\cdot 1\cdot\sin\left(\pi-\frac{5\pi}{14}\right)$$

$$\cdot\sin\left(\pi-\frac{3\pi}{14}\right)\sin\left(\pi-\frac{\pi}{14}\right)$$

$$=\sin^2\frac{\pi}{14}\cdot\sin^2\frac{3\pi}{14}\cdot\sin^2\frac{5\pi}{14}$$

$$=\left\{\sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\right)^2$$

$$=\left\{\sin\left(\frac{\pi}{2}-\frac{3\pi}{7}\right)\cdot\sin\left(\frac{\pi}{2}-\frac{2\pi}{7}\right)\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)\right\}^2$$

$$=\left\{\cos\frac{3\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}\right\}^2$$

$$=\left\{\cos\left(\pi-\frac{4\pi}{7}\right)\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}\right\}^2$$

$$=\left\{-\cos\frac{4\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7}\right\}^2$$

$$=\left\{-\left(-\frac{1}{8}\right)\right\}^2=\frac{1}{64} \quad [fromEq.(i)]$$

Question24

If $f(\theta)=\cos^3\theta+\cos^3\left(\frac{2\pi}{3}+\theta\right)+\cos^3\left(\theta-\frac{2\pi}{3}\right)$, then $f\left(\frac{\pi}{5}\right)=$

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Options:

A. $\frac{3(\sqrt{5}-1)}{16}$

B. $\frac{3\sqrt{10-2\sqrt{5}}}{8}$

C. $\frac{3\cdot\sqrt{10+2\sqrt{5}}}{8}$

D. $\frac{3(\sqrt{5}+1)}{16}$

Answer: D

Solution:

$$
\begin{aligned}
f(\theta) =&\cos^3\theta+\cos^3\left(\frac{2\pi}{3}+\theta\right)+\cos^3\left(\theta-\frac{2\pi}{3}\right) \\
&=\frac{1}{4}\{\cos 3\theta+3\cos\theta\} \\
&\quad+\frac{1}{4}\left\{\cos(2\pi+3\theta)+3\cos\left(\frac{2\pi}{3}+\theta\right)\right\} \\
&\quad+\frac{1}{4}\left\{\cos(2\pi-3\theta)+3\cos\left(\frac{2\pi}{3}-\theta\right)\right\}: \\
&=\frac{3}{4}\cos 3\theta+\frac{3}{4}\left\{\cos\theta+\cos\left(\frac{2\pi}{3}+\theta\right)\quad+\cos\left(\frac{2\pi}{3}-\theta\right)\right\} \\
&=\frac{3}{4}\cos 3\theta+\frac{3}{4}\left\{\cos\theta+2\cdot\cos\frac{2\pi}{3}\cdot\cos\theta\right\}
\end{aligned}
$$

$$
\begin{aligned}
&=\frac{3}{4}\cos 3\theta+\frac{3}{4}\left\{\cos\theta+2\left(-\frac{1}{2}\right)\cdot\cos\theta\right\} \\
&=\frac{3}{4}\cos 3\theta+\frac{3}{4}\{\cos\theta-\cos\theta\} \\
&\therefore f(\theta)=\frac{3}{4}\cos 3\theta \\
&\text{Now, } f\left(\frac{\pi}{5}\right)=\frac{3}{4}\cos\frac{3\pi}{5}=\frac{3(\sqrt{5}+1)}{16}
\end{aligned}
$$

Question25

For $0 \leq x \leq \pi$, if $81^{\sin^2 x}+81^{\cos^2 x}=30$, then $x=$

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Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{15}$

D. $\frac{\pi}{8}$

Answer: A

Solution:

To solve the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ for $0 \leq x \leq \pi$, we can follow these steps:

Start with the given equation:

$81^{\sin^2 x} + 81^{\cos^2 x} = 30$

Recall the identity $\cos^2 x = 1 - \sin^2 x$ and substitute into the equation:

$81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$

Express the second term as a fraction:

$81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$

Let $t = 81^{\sin^2 x}$, then the equation becomes:

$t + \frac{81}{t} = 30$

Multiply through by t to clear the fraction:

$t^2 - 30t + 81 = 0$

Solve this quadratic equation using the quadratic formula $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$t = \frac{30 \pm \sqrt{30^2 - 4 \cdot 1 \cdot 81}}{2 \cdot 1}$

Simplify:

$t = \frac{30 \pm \sqrt{900-324}}{2}$

$t = \frac{30 \pm \sqrt{576}}{2}$

$t = \frac{30 \pm 24}{2}$

Thus, the solutions for t are:

$t = 27 \quad \text{or} \quad t = 3$

Translate back to $\sin^2 x$ using $t = 81^{\sin^2 x}$:

For $t = 27$:

$81^{\sin^2 x} = 3^3 \Rightarrow 3^{4\sin^2 x} = 3^3 \Rightarrow 4\sin^2 x = 3$

$\sin x = \frac{\sqrt{3}}{2}$

For $t = 3$:

$81^{\sin^2 x} = 3 \Rightarrow 3^{4\sin^2 x} = 3^1 \Rightarrow 4\sin^2 x = 1$

$\sin x = \frac{1}{2}$

Now find x within the given range $0 \leq x \leq \pi$:

For $\sin x = \frac{\sqrt{3}}{2}$, the solutions are:

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

For $\sin x = \frac{1}{2}$, the solutions are:

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Thus, the values of x that satisfy the original equation are:

$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3}$

Question26

$$\frac{(1+\tan 32^\circ)}{(1-\tan 48^\circ)} =$$

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Options:

A.

1

B.

2

C.

3

D.

4

Answer: D

Solution:

Let's evaluate the expression exactly as shown:

$$\frac{1+\tan 32^\circ}{1-\tan 48^\circ}$$

1. Direct numerical evaluation

$$\tan 32^\circ \approx 0.6249, \quad \tan 48^\circ \approx 1.1106$$

So,

$$\text{numerator} = 1 + 0.6249 = 1.6249$$

$$\text{denominator} = 1 - 1.1106 = -0.1106$$

$$\frac{1.6249}{-0.1106} \approx -14.7$$

2. Compare with options:

- A: 1
- B: 2
- C: 3
- D: 4

The value -14.7 is not close to any of the given choices.

---

3. Likely intended expression

A common identity is:

$$\frac{1+\tan x}{1-\tan x} = \tan(45^\circ + x)$$

If the denominator were $1 - \tan 32^\circ$ (not 48°), then:

$$\tan(45^\circ + 32^\circ) = \tan 77^\circ \approx 4.33$$

Which is closest to **4 (option D)**.

Question27

$\sin\alpha + \cos\alpha = m \Rightarrow \sin^6\alpha + \cos^6\alpha =$

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Options:

A. $\frac{4+3\left(m^2-1\right)^2}{4}$

B. $\frac{4-3\left(m^2-1\right)^2}{4}$

C. $\frac{3+4\left(m^2-1\right)^2}{4}$

D. $\frac{4-3\left(m^2+1\right)^2}{4}$

Answer: B

Solution:

Given, $\sin\alpha + \cos\alpha = m$

$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha = m^2$

$\sin\alpha\cos\alpha = \frac{m^2-1}{2}$

$\sin^6\alpha + \cos^6\alpha = \left(\sin^2\alpha\right)^3 + \left(\cos^2\alpha\right)^3$

$= \left(\sin^2\alpha + \cos^2\alpha\right)\left(\sin^4\alpha + \cos^4\alpha\right.$

$\left. - \sin^2\alpha\cos^2\alpha\right)$

$= \left(\sin^2\alpha\right)^2 + \left(\cos^2\alpha\right)^2 + 2\sin^2\alpha\cos^2\alpha$

$- 2\sin^2\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha$

$= \left(\sin^2\alpha + \cos^2\alpha\right)^2 - 3\sin^2\alpha\cos^2\alpha$

$= 1 - \frac{3\left(m^2-1\right)^2}{4}$

$= \frac{4-3\left(m^2-1\right)^2}{4}$
